Simulation of a single mode laser system with periodically modulated signal

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Abstract: The intensity correlation function $C(t)$ of the single mode laser system was calculated by using the linear approximation method, the time evolution of $C(t)$ was researched as well as the influence of modulation signal frequency Ω and amplitude B on $C(t)$ was analyzed in detail. In order to observe the relationship between the output light intensity and time directly, the Simulink software is used to simulate the system reasonably, and the agreement between the simulation results and the simulation results is analyzed. It is found that the evolution of the theoretical results and the simulation results are basically consistent, and there is a process of periodic oscillation attenuation in the curse of $C(t)$.

1. Introduction

The correlation function of light intensity is a basic statistical physical quantity to describe the dynamic properties of laser system. The relative fluctuation of power spectrum, output power, correlation time and intensity of laser system can be derived from the correlation function of light intensity. Stochastic resonance phenomenon of laser system can be further studied, which provides a theoretical basis for optimizing the output of laser system, so it has a certain application background in laser communication technology. In addition, by studying the correlation between noise and noise in the evolution of intensity correlation function with time, we can find out the influence of noise and its correlation form on the nonlinear dynamic behaviour of laser system. Therefore, the study of intensity correlation function has certain theoretical significance.

According to the current research situation, there are many difficulties in the study of single-mode laser system, especially in the study of single-mode laser intensity correlation function and chaotic system. Due to the lack of programming ability in numerical simulation experiments, the study of single-mode laser system and its chaos is restricted greatly. This limitation is also an important factor that hinders the further accuracy of the research results. Based on the wide application of MATLAB software platform, Simulink in the simulation software has been developed and established in scientific, academic and industrial fields. The working process of the visual model in the software presents an interactive form, so the researchers can change the specific values of the parameters in the system at any time.

Based on the single-mode laser gain model under the action of periodic modulation signal, the intensity correlation function reflecting the dynamic properties of the laser is studied, and the evolution of the intensity correlation function with time is discussed. The influence of amplitude and frequency of modulated signal on the correlation function of light intensity with time is analyzed in detail. According to the results of theoretical analysis, the Simulink software is used to simulate the system reasonably. By comparing the theoretical results with the simulation results, it is found that the two output results are basically consistent with the time evolution process, and both appear periodic oscillation attenuation with time.

2. Model Construction

The Langevin equation of single-mode laser gain model is

$$\frac{dI}{dt}=2y_0\sqrt{I}-2I-\frac{4cI}{1+I}-\frac{4I}{1+I}\xi(t)+2\sqrt{I}\eta(t) \tag{1}$$

Using periodic signal $B\cos\Omega t$ to modulate pump noise, then the form of equation (1) is

$$\frac{dI}{dt}=2y_0\sqrt{I}-2I-\frac{4cI}{1+I}-\frac{4I}{1+I}\xi(t)B\cos\Omega t+2\sqrt{I}\eta(t) \tag{2}$$

where pump noise and quantum noise satisfy the following statistical properties

$$\begin{gathered}\langle\xi(t)\rangle=\langle\eta(t)\rangle=0\\ \langle\xi(t)\eta(s)\rangle=\langle\xi(s)\eta(t)\rangle=0\\ <\xi(t)\xi(s)>=\frac{P}{2\tau}e^{-\frac{|s-s'|}{\tau}},<\eta(t)\eta(s)>=Q\delta(s-s')\end{gathered} \tag{3}$$

where I、B、Ω represent the light intensity, amplitude of modulated signal and frequency of modulated signal respectively in the equation (1)-(3)，P、Q represent pump noise and quantum noise intensity respectively, τ is the auto-correlation time of pump noise, $\xi(t)$ is the colour pump noise，$\eta(t)$ is the quantum noise.

Writing $I=I_0+\delta(t)$, we can obtain

$$\frac{d\delta(t)}{dt}=-\gamma\delta(t)-\frac{4I_0}{1+I_0}\xi(t)B\cos\Omega t+2\sqrt{I_0}\eta(t)+M$$

(4)

where $\gamma=2+\frac{4c}{1+I_0}$, $M=2y_0\sqrt{I_0}-2I_0-\frac{4cI_0}{1+I_0}$

The formula (3) is replaced by the formula (4)

$$\delta(t)=e^{-\gamma t}\left[\frac{2y_0\sqrt{I_0}-2I_0-\frac{4cI_0}{1+I_0}}{\gamma}(e^{\gamma t}-1)-\frac{4I_0}{1+I_0}\int_0^t\xi(s)e^{\gamma s}B\cos\Omega s ds+2\sqrt{I_0}\int_0^t\eta(s)e^{\gamma s}ds\right] \tag{5}$$

$$\delta(t+t')=e^{-\gamma(t+t')}\left[\frac{M}{\gamma}(e^{(t+t')}-1)+\frac{4I_0}{1+I_0}\int_0^{t+t'}\xi(s')e^{\gamma s'}B\cos\Omega s'ds'+2\sqrt{I_0}\int_0^{t'}\eta(s')e^{\gamma s'}ds'\right] \tag{6}$$

According to the definition of steady-state intensity correlation function

$$C(t)=\lim_{t'\to\infty}\overline{\langle I(t')I(t'+t)\rangle}=\lim_{t'\to\infty}(\frac{\Omega}{2\pi}\int_{t'}^{t'+2\pi/\Omega}\langle I(t')I(t'+t)\rangle dt')$$

(7)

Putting (5) and (6) into (7), then get

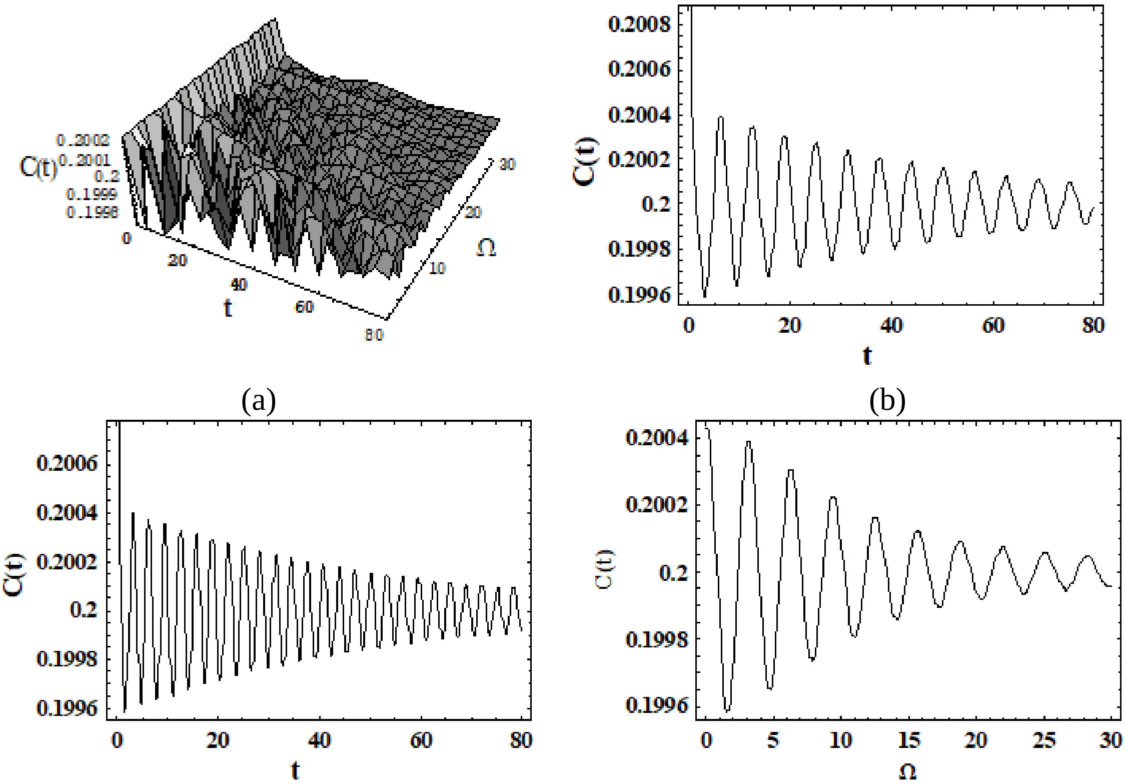
$$C(t)=I_0{}^2+\frac{2M}{\gamma}I_0+\left(\frac{M}{\gamma}\right)^2+\Big(\frac{2I_0Q}{\gamma}+\frac{2\gamma k_2PI_0{}^2B^2}{\tau(1+I_0)^2(k_2{}^2+\Omega^2)(\gamma^2+\Omega^2)}$$
$$+\frac{2k_2PI_0{}^2B^2\Omega^2}{\tau\gamma(1+I_0)^2(k_2{}^2+\Omega^2)(\gamma^2+\Omega^2)}-\frac{2k_1\gamma PI_0{}^2B^2}{\tau(1+I_0)^2(k_1{}^2+\Omega^2)(\gamma^2+\Omega^2)}$$
$$-\frac{2k_1PI_0{}^2B^2\Omega^2}{\tau\gamma(1+I_0)^2(k_1{}^2+\Omega^2)(\gamma^2+\Omega^2)}\Big)e^{-\gamma|t|}+\frac{4P(\gamma^2-\tau^{-2}+\Omega^2)I_0{}^2B^2\Omega^2\cos\Omega t}{\tau(1+I_0)^2(k_1{}^2+\Omega^2)(k_2{}^2+\Omega^2)}e^{-\frac{|t|}{\tau}}$$
$$+\frac{8\Omega PI_0{}^2B^2e^{-\frac{|t|}{\tau}}}{\tau^2(1+I_0)^2(k_1{}^2+\Omega^2)(k_2{}^2+\Omega^2)}\sin\Omega|t|$$

(8)

where $\kappa_1=\gamma-\tau^{-1}$, $\kappa_2=\gamma+\tau^{-1}$.

It can be seen from the above formula that the correlation function of light intensity has two characteristics, one is that because the pump noise is coloured noise, so that the steady-state mean intensity correlation function $C(t)$ has two time scales; the other is that the signal modulates the pump noise so that it is a periodic function of the modulation signal frequency. The influence of signal amplitude and frequency on the evolution of the signal over time is discussed according to equation (8).

3. The influence of modulated signal frequency on the evolution of intensity correlation function



(a) (b) (c) (d)

Fig. 1 Time evolution of the $C(t)$ for different values of Ω with the parameters

$$B = 20, \ c = 12, \tau = 50, I_0 = 5, P = 0.002, Q = 0.002, y_0 = 1$$

Fig.1 shows the influence of modulated signal frequency on the evolution of intensity correlation function, and Fig.1(a) shows 3D stereograms which reflects the effect of modulated signal frequency Ω on the evolution of modulation signal over time, the section of the curve is shown in fig. 1(b)-(d). Because of the influence of Ω, the evolution of $C(t)$ is more complicated. The sectional drawing of $C(t)$ shows that the curve exhibits periodic oscillation attenuation, and it is found that, at a given time t, the curve of $C(t)-\Omega$ varies periodically with Ω. It can be found that the frequency of the periodic oscillation increases, but the amplitude decreases from Fig.1(b)-(d). In other words, the attenuation of the intensity function accelerated. When the evolution time is long enough, the intensity correlation function values of periodic oscillation attenuation with different amplitudes and frequencies in Figure 2 tend to be consistent.

In the case of $\tau >> 1$, the evolution of $C(t)$ with time is periodic oscillatory attenuation regardless of the size of Ω. Obviously, the modulation of periodic signal to coloured pump noise is the main reason for the evolution of intensity correlation function with time, which indicates that the interaction between modulation signal and pump noise "colour" has a great influence on the dynamic behaviour of laser system.

4. Research on Visualization Model and its Simulation characteristics

In recent years, a large number of researchers have studied the single-mode laser system deeply, and the single-mode laser system has become an important way to study the laser. In a single-mode laser system, the laser intensity correlation time is an important statistic to describe the laser system, and it is of great significance to study the dynamic characteristics of the laser system. In many scientific research work, the laser intensity correlation time can be used to measure the specific value of different scientific research methods in different research problems, so it has a higher value for practical and scientific research. But the problem that can't be ignored is that, there are many difficulties in calculation and analysis of single mode laser system like other optical systems in theoretical research. In view of this, researchers usually use related software to simulate the system by numerical experiments. Simulink, a simulation software based on MATLAB platform, is playing an important role in the field of scientific research. The software can be used to visualize and simulate the parameters of the laser system, and if the researchers change the parameters of the model, the corresponding simulation results can be observed directly.

The main components of a single mode laser are the resonant cavity and the active medium, and the formation of the light is realized by two parts of the spontaneous emission and stimulated radiation of the atom. In the analysis of such systems, the "semiclassical" theory is usually used to derive the Lorenz equation that reflects the dynamic behavior of a single mode laser system. Maxwell-Bloch equation is used as the theoretical basis for describing uniform widening single mode laser in the early stage of research. Until 1975 Haken et.al simplified it to the following nonlinear equations by coordinate transformation

$$\frac{dE}{dt} = -\kappa(E + P) \tag{11}$$

$$\frac{dP}{dt} = \gamma_{\perp}\kappa(ED - P) \tag{12}$$

$$\frac{dD}{dt} = \gamma_{//}(\lambda + 1 - D - \lambda EP) \tag{13}$$

The equation includes the amplitude of the intracavity light field E; t represents the time; κ is the attenuation rate of the optical field in the cavity; P is the amplitude of atomic polarization; $\lambda_{\perp}$ represents the attenuation rate of the polarization intensity; D is the inversion of the number of particles; $\lambda_{//}$ is the attenuation rate of the number of particles; and λ represents the pump intensity. The famous Lorenz model corresponding to the formula (1)-(3) read

$$\frac{dX}{dt} = \sigma(Y - X) \tag{14}$$

$$\frac{dY}{dt} = -XZ - rX - Y \tag{15}$$

$$\frac{dZ}{dt} = XY - bZ \tag{16}$$

It is found that the above two sets of equations are identical in form through observation and analysis. Here Y is proportional to the horizontal temperature change; Z is proportional to the vertical temperature change; σ is Prandtl number; b represents the geometric factor; and γ represents the Rayleigh number. Thus the single mode laser system can be studied by (14)-(16), and each quantity in (14)-(16) is reduced dimensionless. There are two nonlinear terms XY and XZ in the system, so it's a nonlinear system. The equations of many real physical systems can also be transformed into these forms, so the system has a certain universal significance. In laser system, if the relation between Q value of optical resonator and the width of the spectral line of longitudinal mode is known, then the light intensity at any time is given as following equation

$$I(t) = I_0 e^{-t/\tau_R} \tag{17}$$

The amplitude of the light intensity is

$$A(t) = A_0 e^{-t/\tau_R} \tag{18}$$

The intensity of light can be expressed as

$$u(t) = A(t)e^{-i\omega t} = A_0 e^{-t/\tau_R} \cdot e^{i\omega t} \tag{19}$$

From the above formulas (17)-(19), we can see that there is a certain relationship between the amplitude of light field and the intensity of light in the cavity of laser system, so we can establish an integrated simulation environment for laser system from visual modeling to visual analysis by using the simulation software Simulink. The modeling strategy is described as follows: in the first step, the theoretical model of (14)-(16) expression is decomposed into different functional units, such as addition, subtraction, multiplication and micro (product) division. Ensuring the Simulink module library is opened in Matlab software, and the corresponding modules of each functional unit are selected from them. The display unit used to observe the waveform of each component signal with oscilloscope can be replaced by scope module, and the planar graphic display unit which can be used to display the phase diagram of dynamic system can be represented by Graph module. In the second step, according to the relationship of the operation units in the theoretical model, the appropriate logic relation is selected, and the modules are dragged and connected to construct the simulation system that meets the requirements. The third step, setting the parameters of different physical quantities in the module by clicking on the properties of each module. Attributed to the convenience of Simulink software, the whole process of building a visual model is more convenient. Finally, the visual model of single-mode laser system is shown in figure 2.

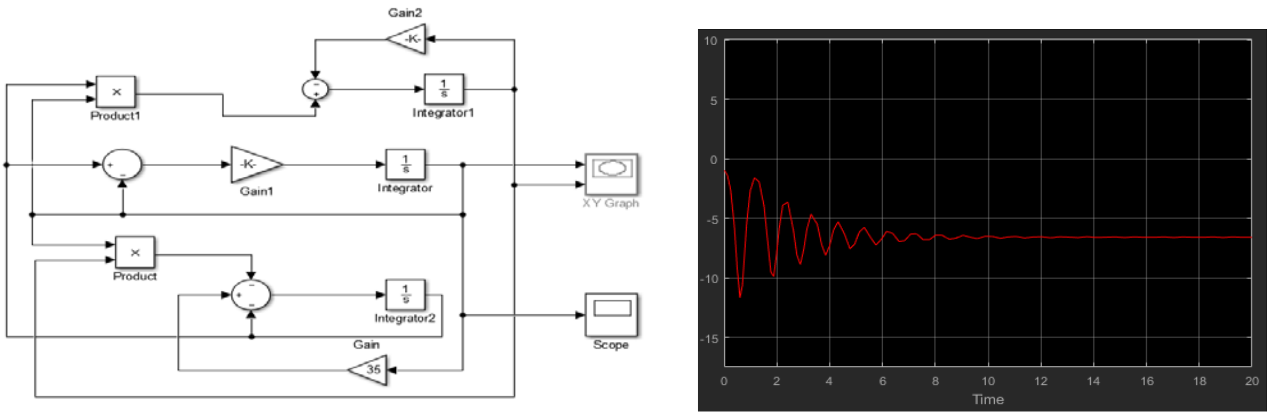


Fig.2 Visual simulation system of single-mode laser system

The system parameters are settled and the output square in the single mode laser system are analyzed by using the simulation system. It can be seen that the variation of light field X in Fig.2 is decaying with periodic oscillation from the simulation results. Because of the function relation in formula (17)-(19), the amplitude of transverse axis and the period of longitudinal axis in Fig.2 are not completely consistent with Fig.1. However, comparing with Fig.2, we can find that the variation trend of the output intensity of the single mode laser system is basically the same as that of the previous single-mode laser system in figure 1(b)-(d), it means that the theoretical calculation results are in good agreement with the simulation results.

5. Conclusions

Combined with the above discussion, it can be found that the theoretical results are consistent with the simulation results, and the results are as follows:

(1)When the frequency of modulation signal increases, the evolution of the modulation signal with time is periodic oscillation attenuation.

(2) The linearization approximation method is applied in the calculation of the light intensity correlation function, and it shows that if the normalized steady-state mean relative intensity fluctuation $C(0) << 1$, the linearization approximation method is reliable, and the range of parameters selected in this paper can meet the requirement of linearization approximation. Therefore, the research in this paper has some significance for studying the effect of noise on laser dynamics.

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